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A SLIDE RULE FOR DETERMINING 10,000-FOOT PRESSURE

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[Weather Bureau Airport station, Seattle, Wash., March 1940]

One of the many problems confronting meteorologists engaged in airway forecasting arises from the frequent lack, during adverse weather, of current wind-aloft data. It is during such periods that accurate data concerning upper-air circulation are most important in properly planning instrument flights.

Some excellent work has recently been done by Vernon and Ashburn,[1] and by Haynes [2] on methods of computing winds aloft when actual observations are missing.

It is felt that the method now employed at the Seattle Airport Station of the Weather Bureau may be used to advantage in regions where there is a scarcity of reports of winds and/or pressures aloft, and in particular, when there is a limited amount of time available for such determinations.

At Seattle a chart of 10,000-foot pressures is constructed daily for the United States, western Canada, Alaska, and, as far as ship reports are available, the section of the Pacific Ocean adjoining the Pacific coast.

A network of radiosonde observations is available from the United States and Alaska, and an airplane sounding is received from Edmonton, Alberta, Canada. These reports do not normally provide sufficient information for construction of an accurate upper-air map along the immediate Pacific coast line, and particularly between Seattle and Juneau.

In order to provide a close network of pressure values, it has been the practice at Seattle during the past two years to estimate upper-air temperatures at a number of coastal and Canadian stations and ships in the adjacent Pacific Ocean, and use the reported sea-level pressures to obtain the 10,000-foot pressures.

The pressure reduction may be accomplished by means of various tables available; however, it is the purpose of this paper to describe the construction of a simple slide rule which is found very convenient for the purpose.

The hypsometric equation may be written:

$$z=\frac{RT_v}{Mg}\log_{10}\frac{P_0}{P} \tag{1}$$

where:

z=Difference of height, in centimeters, between upper and lower station.

$R=2.8703\times10^6$.

T_v=Absolute virtual temperature (centigrade).

$=(1+.605q)\ T=\left(1+\frac{.376e}{P}\right)T$, Approx.

in which q=Specific humidity, $\frac{0.6221e}{P-38e}$, e=Vapor pressure, mb., and T=Absolute centigrade temperature. M=Modulus of common logarithms=0.434294. g=Acceleration of gravity, c.g.s. units, average value (with respect to height) between upper and lower levels. P_0=Pressure at lower level measured in any units, provided P_0 and P are in same units; P_0 will here be used for sea-level pressure. P=Pressure at higher level.

If, instead of constructing our pressure map for a surface of equal geometric height, we construct it for a surface of equal geopotential, the gradient or geostrophic wind equations for motion within this surface will apply more exactly. By definition it is only within a surface of equal geopotential that no work is done against gravity, since the average value of gravity from sea-level up to the surface of equal geopotential will always be the same.

With g and z defined exactly as in the hypsometric equation, and the lower point taken at sea-level, geopotential may be defined as follows:[3] geopotential$=gz$, (g=average value).

We may express geopotential in terms of dynamic meters, defined (after V. Bjerknes) as

$$Z_d=\text{height, dynamic meters}=Z\ \frac{g\text{(average)}}{1{,}000}$$

where Z =height, geometric meters.

In radiosonde observational work, the Weather Bureau uses as a unit of height 0.98 dynamic meter, which is equivalent to exactly 1 geometric meter when average g=980 dynes.

To adopt a unit of 0.98 dynamic meter, it is merely necessary to substitute a value of 980 for g in the hypsometric equation.

Substituting for R, M, and g, the hypsometric equation becomes:

$$\frac{Z_d}{.98}=Z\text{ (meters)}=67.439\ T_v\log_{10}\frac{P_0}{P}. \tag{2}$$

At 10,000 feet (3,048 geometric meters),

$$\log P-\log P_0=-\frac{1}{.022126T_v}. \tag{3}$$

Since the average value of gravity from sea-level to 10,000 feet=980 dynes at approximately 38°35′ latitude, the surface will be exactly 10,000 geometric feet (3,048 meters) at this latitude. Geometric height will be less toward the Poles and greater toward the Equator.

From an inspection of equation (3) it is evident that, if log P and log P_0 are plotted on the same logarithmic scale, the difference between any two corresponding

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[1] Vernon, Edward M., and Ashburn, Edward V. A practical method for computing winds aloft from pressure and temperature fields. MONTHLY WEATHER REVIEW, September 1938, 66: 267–274.

[2] Haynes, B. C. Upper-wind forecasting. MONTHLY WEATHER REVIEW, January 1938, 66: 4–6.

[3] See for instance, *Smithsonian Meteorological Tables*, Fifth Revised Edition, pp. LIII–LV.

values of log P and log P_0, as measured linearly along the scale, will be a function of T_v only. For any given value of T_v, the difference log $P-$log P_0 will be constant.

However, a plot of P and P_0 on an extended log scale would either be inconveniently long, or the consecutive values of pressure too crowded for practical use. The following arbitrary changes reduce the scale to a practical size, using inches for linear measure:

$$(100 \log P-277)-(100 \log P_0-300)=23-\frac{100}{.022126 T_v}. \quad (4)$$

Suppose we label the terms as follows:

"A" scale$=100 \log P_0-300$,
"B" scale$=100 \log P-277$,
"C" scale$=23-\dfrac{100}{.022126\ T_v}$.

Arbitrary changes made in the equation above give the "A" and "B" scales a common zero point at 1,000 mb. on the "A" scale. The "A" and "B" scales are measured to the right, for convenience, of the common zero point along the same straight line, except that, on the "A" scale, values of pressure below 1,000 mb. are negative, and must be measured to the left.

The "A" and "B" scales constitute the stationary part of the rule. The "C" scale is the sliding portion, and is measured to the right of its own zero point.

Equation (4) is in a form convenient for use with pressure measured in millibars, and linear distance along the scale in inches. If it is desired to use other units, corresponding changes may be made. With the units here used the rule will be about 14 inches long.

In tables 1, 2, 3, the values of the "A", "B," and "C" scales are given.

TABLE 1.—*"A" scale, $A=100 \log P_0-300$*

[P_0=sea-level pressure, mb.]

P_0	A	P_0	A	P_0	A
	Inches		*Inches*		*Inches*
948	−2.319	984	−0.700	1018	0.775
950	−2.228	986	−.612	1020	.860
952	−2.136	988	−.524	1022	.945
954	−2.045	990	−.436	1024	1.030
956	−1.954	992	−.349	1026	1.115
958	−1.863	994	−.261	1028	1.199
960	−1.773	996	−.174	1030	1.284
962	−1.682	998	−.087	1032	1.368
964	−1.592	1000	.000	1034	1.452
966	−1.502	1002	.087	1036	1.536
968	−1.412	1004	.173	1038	1.620
970	−1.323	1006	.260	1040	1.703
972	−1.233	1008	.346	1042	1.787
974	−1.144	1010	.432	1044	1.870
976	−1.055	1012	.518	1046	1.953
978	−.966	1014	.604	1048	2.036
980	−.877	1016	.689	1050	2.109
982	−.789				

TABLE 2.—*"B" scale, $B=100 \log P-277$*

[P=pressure at 10,000 feet, mb.]

P	B	P	B	P	B
	Inches		*Inches*		*Inches*
622	2.379	666	5.347	710	8.126
624	2.518	668	5.478	712	8.248
626	2.657	670	5.607	714	8.370
628	2.796	672	5.737	716	8.491
630	2.934	674	5.866	718	8.612
632	3.072	676	5.995	720	8.733
634	3.209	678	6.123	722	8.854
636	3.346	680	6.251	724	8.974
638	3.482	682	6.378	726	9.094
640	3.618	684	6.506	728	9.213
642	3.754	686	6.632	730	9.332
644	3.889	688	6.759	732	9.451
646	4.023	690	6.885	734	9.570
648	4.158	692	7.011	736	9.688
650	4.291	694	7.136	738	9.806
652	4.425	696	7.261	740	9.923
654	4.558	698	7.386	742	10.040
656	4.690	700	7.510	744	10.157
658	4.823	702	7.634	746	10.274
660	4.954	704	7.757	748	10.390
662	5.086	706	7.880	750	10.506
664	5.217	708	8.003		

TABLE 3.—*"C" scale; $C=23-\dfrac{100}{.022126\ T_v}$*

T_v	°C.	"C"	T_v	°C.	"C"	T_v	°C.	"C"
		Inches			*Inches*			*Inches*
233	−40	3.603	257	−16	5.414	281	8	6.916
234	−39	3.686	258	−15	5.482	282	9	6.973
235	−38	3.768	259	−14	5.550	283	10	7.030
236	−37	3.849	260	−13	5.617	284	11	7.086
237	−36	3.930	261	−12	5.684	285	12	7.141
238	−35	4.010	262	−11	5.750	286	13	7.197
239	−34	4.090	263	−10	5.815	287	14	7.252
240	−33	4.168	264	−9	5.880	288	15	7.307
241	−32	4.247	265	−8	5.945	289	16	7.361
242	−31	4.324	266	−7	6.009	290	17	7.415
243	−30	4.401	267	−6	6.073	291	18	7.469
244	−29	4.477	268	−5	6.136	292	19	7.522
245	−28	4.553	269	−4	6.198	293	20	7.575
246	−27	4.628	270	−3	6.261	294	21	7.627
247	−26	4.702	271	−2	6.323	295	22	7.679
248	−25	4.776	272	−1	6.384	296	23	7.731
249	−24	4.849	273	0	6.445	297	24	7.783
250	−23	4.922	274	1	6.505	298	25	7.834
251	−22	4.994	275	2	6.565	299	26	7.884
252	−21	5.065	276	3	6.625	300	27	7.934
253	−20	5.136	277	4	6.684	301	28	7.984
254	−19	5.206	278	5	6.742	302	29	8.034
255	−18	5.276	279	6	6.801	303	30	8.084
256	−17	5.345	280	7	6.859			

The accompanying diagram (fig. 1) illustrates the rule. The 10,000-foot pressure calculations are made thereon simply by setting the zero point of the "C" scale on the sea-level pressure, and reading the 10,000-foot pressure opposite the determined mean virtual temperature.

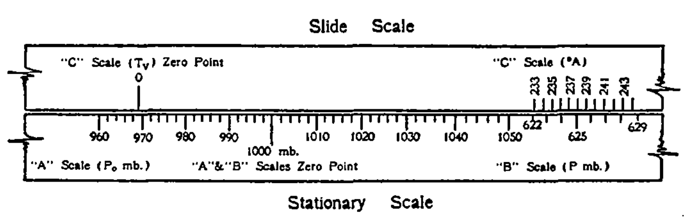

FIGURE 1.—Plan of 10,000-foot slide rule.

In preparing a pressure map it is of course necessary to estimate upper-air temperatures where they are not available from actual observations. This might at first appear difficult. However, surface temperature serves as a starting point for the estimated curve. A careful study of the meteorological factors involved, particularly the history and trajectory of air masses will give a fair picture of temperature conditions aloft. It is especially necessary to check estimates against such actual temperatures as are available when a radiosonde observation falls within the same air mass. With experience, considerable accuracy is possible.

If the pressure map is extended to land areas of any considerable elevation, and the reductions are made from reported sea-level pressure, it becomes necessary in determining T_v to use a fictitious temperature for that portion of the 10,000-foot column which is below the level of the station. This fictitious value is the mean of the current surface temperature and that 12 hours previously. It is that used in reducing station pressure to sea level. On the assumption that reduction to sea level has been made exactly according to this temperature by means of the hypsometric equation, we may without appreciable error construct a temperature curve, which, from sea level to the station level follows the above fictitious temperature, and from the surface level to 10,000 feet, follows the estimated free-air temperatures. The mean of this total curve, with a small correction for water vapor content, gives the value of T_v.

In areas north of Seattle, including the north Pacific, it is usually necessary to apply only slight corrections for moisture content, and as a result, the difference between virtual and actual temperature is small. Using Hann's empirical vapor pressure equation $\left(\log \frac{e}{e_0}=\frac{-Z}{6200}\right)$, the value of average vapor pressure for the 10,000-foot column becomes roughly 0.6 that of the surface vapor pressure. If the dew point is 5° C. (a representative winter value in the north Pacific), the difference between T and T_v is about 0.6° C. (T_v higher than T). In summer the difference may be as much as 1.0° C. or slightly higher. In more southerly latitudes, $T_v - T$ is often considerably greater.

It is recognized that, at times, noticeable error may result in the above pressure determinations where it is difficult to estimate temperatures, but such errors will usually smooth out in drawing isobars on the pressure map. Experience at Seattle in the use of maps so constructed indicates that quite accurate average values of upper winds may be determined from them.

Acknowledgment is due L. P. Harrison of the Weather Bureau Aerological Division for helpful suggestions.

AN UNUSUAL HALO DISPLAY

By D. B. O. SAVILE

[Control Experimental Farm, Department of Agriculture, Ottawa, Ontario, February 1941]

Most of the individual arcs, halos, and parhelia that are associated with an abundance of ice crystals in the atmosphere are not so rare as to merit repeated description, but highly complex displays are far from common. For this reason, and because it seems to throw some light on the precise cause of the sun pillar, the display witnessed at Ottawa, Canada, on January 27, 1941, is worthy of record.

The common 22° halo started to develop before the sun was 3° above the horizon, and was almost continuously distinguishable until sunset. About 10 a. m., E. S. T., the 46° halo became faintly visible to eyes fully adapted to bright light. By 2 p. m. both halos, the horizontal parhelic circle, and the 22° parhelia were all well defined. Developments were then watched from open ground, and notes were taken for some time. During the next hour there were frequent variations in the intensity and extent of some of the components, but those shown in figure 1, and described below, were several times simultaneously visible at approximately 2.30 p. m.

The horizontal parhelic circle, LSM, generally extending about 30° beyond the point of intersection with the 46° halo; occasionally slightly exceeding a semicircle in extent.

The sun pillar, UV, frequently extending about 8° above and below the sun; maximum extent about 10° above and 12° below the sun; scarcely wider at the extremities than at the sun; rare with the sun high in the sky.

Complete 22° halo, ABC; not as bright or as well colored as earlier in the day; the inner edge red-brown.

Upper tangent arc of the 22° halo, DAE; brilliant near point of contact, and better colored than the halo; not distinct to the point where it curves downward.

The 46° halo, GFH; brilliant and strongly colored above the parhelic circle, but faint below and never visible quite to the horizon; both color and brightness generally exceeding those of the small halo during the height of the display.

Circumzenithal arc, JK; taken at first for the contact arc of the large halo—indeed the confusion is often made in print; at solar altitudes between 15° and 25° this arc is practically tangent to the halo and is chiefly distinguished by its brilliant coloring; on this occasion the color sensations predominating were violet, yellow-green, orange, and red; the colors were approximately saturated, and were pure in sharp contrast to the broken colors of the other arcs; generally about 60° of arc distinctly visible, but occasionally slightly more.

The parhelia or mock-suns, P and Q, of the small halo were sometimes extremely brilliant, but the presence of the horizontal circle made their colors indistinct. The extremely rare mock-suns, N and R, of the large halo were distinctly visible several times; they were never brilliant and the horizontal circle rendered both color and extent indefinite; lacking accurate means of measurement, it can only be said that they were in approximately the calculated position several degrees outside the halo. Pernter[1] estimates about seven authentic records of this phenomenon, some early descriptions evidently referring merely to the enhancement of light at the intersection of halo and horizontal circle. The counter-sun, T, was visible for a short time as a diffuse light patch, too inconspicuous to be seen by anyone not looking for it.

[1] Pernter, J. M., *Meteorologische Optik*, Dritter Abschnitt. 1902.